21222

3 Hours / 70 Marks   Seat No.

15 minutes extra for each hour

Instructions – (1) All Questions are *Compulsory*.

(2) Answer each next main Question on a new page.

(3) Illustrate your answers with neat sketches wherever necessary.

(4) Figures to the right indicate full marks.

(5) Assume suitable data, if necessary.

(6) Use of Non-programmable Electronic Pocket Calculator is permissible.

(7) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

Marks

1.   **Solve any FIVE of the following:**   10

a)   If $f(x) = x^3 - 3x^2 + 5$, find $f(0) + f(2)$.

b)   Show that $f(x) = 4x^4 + 3 \cos x + x \sin x + 1$ is an even function.

c)   Find $\dfrac{dy}{dx}$ if $y = e^x \cdot \sin x$.

d)   Evaluate $\displaystyle\int \dfrac{1}{3x+7}\, dx$

e)   Evaluate $\displaystyle\int \cos^2 x\, dx$

f)   Find the area under the curve $y = x^2$ from $x = 0$ to $x = 3$ with x-axis.

g)   State the Trapezoidal rule of numerical integration.

P.T.O.

Marks

2.   **Solve any THREE of the following:**   **12**

a) Find $\dfrac{dy}{dx}$ if $x\ \sin y + y\ \sin x = 0$

b) If $x = a(\theta - \sin\theta),\ \ y = a(1 - \cos\theta)$ find $\dfrac{dy}{dx}$

c) A manufacturer can sell x items at price of ₹ $(330 - x)$ each. The cost of producing x items is ₹ $(x^2 + 10x + 12)$. How many items must be sold so that his profit is maximum.

d) Find the radius of curvature for $y = x^3 + 3x^2 + 2$ at $(1,\ 2)$.

3.   **Solve any THREE of the following:**   **12**

a) Find the equation of tangent to the curve $2x^2 - xy + 3y^2 = 18$ at $(3,\ 1)$.

b) Find $\dfrac{dy}{dx}$ if $y = x^{\sin x} + (\tan x)^x$

c) Find $\dfrac{dy}{dx}$ if $y = \log(xe^x)$

d) Evaluate $\displaystyle\int \dfrac{\cos\sqrt{x}}{\sqrt{x}}\,dx$

4.   **Solve any THREE of the following:**   **12**

a) Evaluate $\displaystyle\int \dfrac{1}{x^2 + 3x + 2}\,dx$

b) Evaluate $\displaystyle\int \dfrac{1}{5 + 4\cos x}\,dx$

c) Evaluate $\displaystyle\int \dfrac{1 - \tan x}{1 + \tan x}\,dx$

d) Evaluate $\displaystyle\int \dfrac{\log x}{x(\log x + 2)(\log x + 3)}\,dx$

e) Evaluate $\displaystyle\int_{0}^{7} \dfrac{\sqrt[3]{x}}{\sqrt[3]{x} + \sqrt[3]{7-x}}\,dx$

5.   **Solve any __TWO__ of the following:**   12

   a)   Find the area of the circle $x^2 + y^2 = 25$ by using definite integration.

   b)   Attempt the following:

     i)   Find the order and degree of the D.E.

$$\frac{d^2y}{dx^2} = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}$$

     ii)   Solve the D.E.

$$x\sqrt{1-y^2}\ dx + y\sqrt{1-x^2}\ dy = 0$$

   c)   The velocity of a particle is given by $V = t^2 - 6t + 7$. Find distance covered in 3 seconds.

6.   **Solve any __TWO__ of the following:**   12

   a)   i)   Using Trapezoidal rule calculate the approximate value of $\int_0^4 e^x\ dx$ from given data:

x	0	1	2	3	4
y	1	2.72	7.39	20.09	54.60

     ii)   Evaluate $\int_0^1 \dfrac{dx}{1+x^2}\ dx$ using Simpson's one third rule given by

x	0	0.25	0.5	0.75	1
y	1	0.9412	0.8	0.64	0.5

   b)   Evaluate $\int_0^6 \dfrac{1}{1+x}\ dx$ taking h = 1 by using Simpson's one third rule.

   c)   Evaluate $\int_0^\pi \sin x\ dx$ using Simpson's $\frac{3}{8}$th rule. Divide the interval $[0, \pi]$ into 6 equal parts.

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